01
Introduction to Digital Logic

ENGR 3410 – Computer Architecture
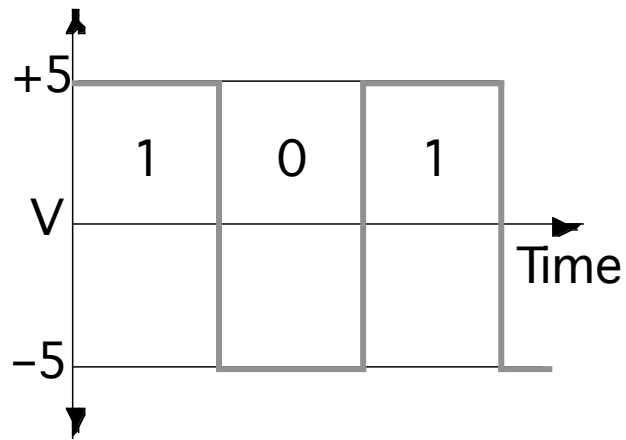
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Acknowledgements

- Patterson & Hennessy: Book & Lecture Notes
- Patterson's 1997 course notes (U.C. Berkeley CS 152, 1997)
- Tom Fountain 2000 course notes (Stanford EE182)
- Michael Wahl 2000 lecture notes (U. of Siegen CS 3339)
- Ben Dugan 2001 lecture notes (UW-CSE 378)
- Professor Scott Hauck lecture notes (UW EE 471)
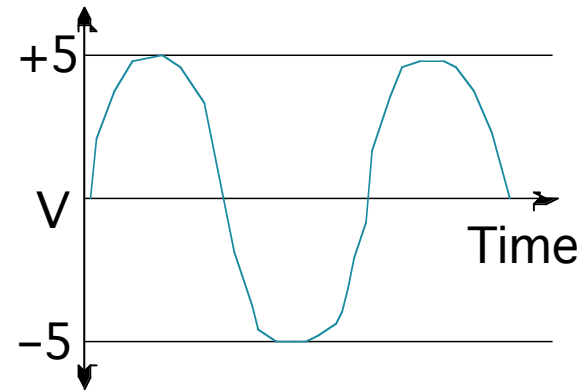- Mark L. Chang lecture notes for Digital Logic (NWU B01)

Example: Car Electronics

- Door ajar light (driver door, passenger door):

- High-beam indicator (lights, high beam selected):

- Seat Belt Light (driver belt in):

- Seat Belt Light (driver belt in, passenger belt in, passenger present):

Basic Logic Gates

- AND / OR / NOT

Digital vs. Analog

Digital:
    only assumes discrete values
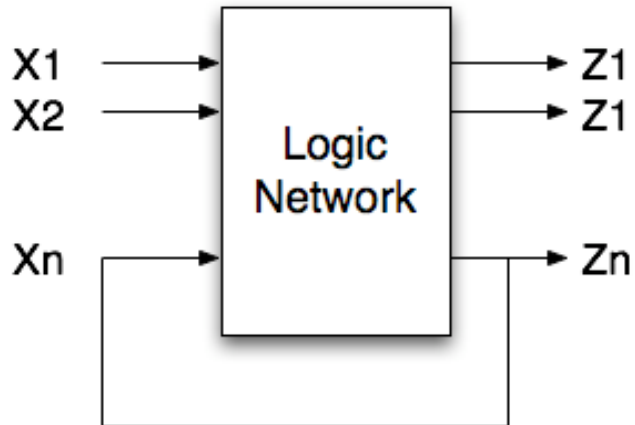
Analog:
    values vary over a broad range
    continuously
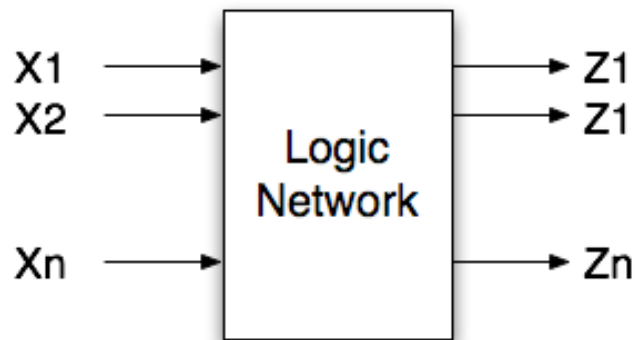
Advantages of Digital Circuits

Combinational vs. Sequential Logic

Sequential logic

X1 → Logic Network → Z1
X2 → → Z1

Xn → → Zn

Network implemented from logic gates. The presence of feedback distinguishes between *sequential* and *combinational* networks.

Combinational logic

X1 → Logic Network → Z1
X2 → → Z1

Xn → → Zn
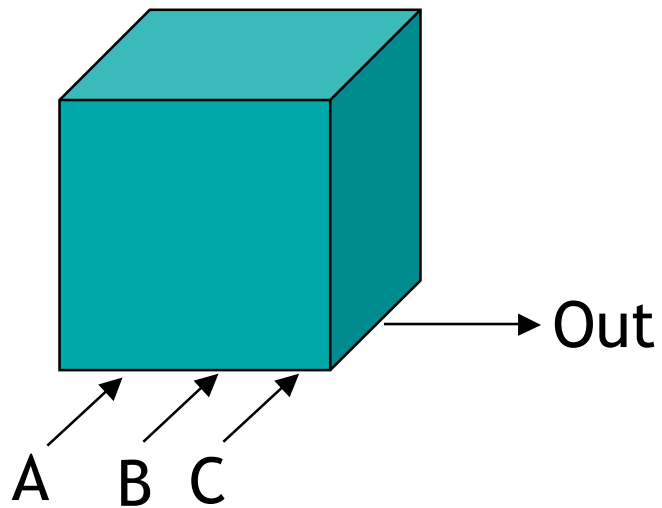
No feedback among inputs and outputs. Outputs are a function of the inputs only.

Black Box (Majority)

- Given a design problem, first determine the function
- Consider the unknown combination circuit a "black box"

Truth Table

Out

A B C

"Black Box" Design & Truth Tables

- Given an idea of a desired circuit, implement it
  - Example: Odd parity - inputs: A, B, C, output: Out

Truth Tables

Algebra: variables, values, operations

In Boolean algebra, the values are the symbols 0 and 1
    If a logic statement is false, it has value 0
    If a logic statement is true, it has value 1

Operations: AND, OR, NOT

X	Y	X AND Y
0	0	0
0	1	0
1	0	0
1	1	1

X	NOT X
0	1
1	0

X	Y	X OR Y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Equations

Boolean Algebra

> values: 0, 1
> variables: A, B, C, . . ., X, Y, Z
> operations: NOT, AND, OR, . . .

NOT X is written as $\overline{X}$
X AND Y is written as X & Y, or sometimes X Y
X OR Y is written as X + Y

Deriving Boolean equations from truth tables:

$$\text{Sum} = \overline{A} B + A \overline{B}$$

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

OR'd together *product* terms
for each truth table
row where the function is 1

if input variable is 0, it appears in
complemented form;
if 1, it appears uncomplemented

Carry = A B

Boolean Algebra

Another example:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$Sum = \overline{A}\ \overline{B}\ Cin + \overline{A}\ B\ \overline{Cin} + A\ \overline{B}\ \overline{Cin} + A\ B\ Cin$$

$$Cout = \overline{A}\ B\ Cin + A\ \overline{B}\ Cin + A\ B\ \overline{Cin} + A\ B\ Cin$$

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Boolean Algebra

Reducing the complexity of Boolean equations

Laws of Boolean algebra can be applied to full adder's carry out function to derive the following simplified expression:

A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

B Cin

A Cin

A B

Cout = A Cin + B Cin + A B

Verify equivalence with the original Carry Out truth table:

place a 1 in each truth table row where the product term is true

each product term in the above equation covers exactly two rows in the truth table; several rows are "covered" by more than one term

Representations of Boolean Functions

- Boolean Function: $F = \overline{X} + YZ$

Truth Table: Circuit Diagram:

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Why Boolean Algebra/Logic Minimization?

Logic Minimization: reduce complexity of the gate level implementation

- reduce number of literals (gate inputs)

- reduce number of gates

- reduce number of levels of gates

fewer inputs implies faster gates in some technologies

fan-ins (number of gate inputs) are limited in some technologies

fewer levels of gates implies reduced signal propagation delays

number of gates (or gate packages) influences manufacturing costs

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Basic Boolean Identities:

- X + 0 =   X * 1 =

- X + 1 =   X * 0 =

- X + X =   X * X =

- X + $\overline{X}$ =   X * $\overline{X}$ =

- $\overline{\overline{X}}$ =

Basic Laws

- Commutative Law:

$X + Y = Y + X$ $\qquad$ $XY = YX$

- Associative Law:

$X+(Y+Z) = (X+Y)+Z$ $\qquad$ $X(YZ)=(XY)Z$

- Distributive Law:

$X(Y+Z) = XY + XZ$ $\qquad$ $X+YZ = (X+Y)(X+Z)$

Boolean Manipulations

- Boolean Function: $F = XYZ + \overline{X}Y + XY\overline{Z}$

Truth Table: Reduce Function:

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Advanced Laws

- $X+XY =$
- $XY + X\overline{Y} =$
- $X+\overline{X}Y =$
- $X(X+Y) =$
- $(X+Y)(X+\overline{Y}) =$
- $X(\overline{\overline{X}}+Y) =$

- Boolean Function: $F = \overline{X}YZ + XZ$

Truth Table: Reduce Function:

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Boolean Manipulations (cont.)

- Boolean Function: $F = (X+\overline{Y}+X\overline{Y})(XY+\overline{X}Z+YZ)$

Truth Table: Reduce Function:

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

DeMorgan's Law

$$\overline{(X + Y)} = \overline{X} * \overline{Y}$$

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X+Y}$	$\overline{X}\cdot\overline{Y}$
0	0	1	1		
0	1	1	0		
1	0	0	1		
1	1	0	0		

$$\overline{(X * Y)} = \overline{X} + \overline{Y}$$

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X\cdot Y}$	$\overline{X}+\overline{Y}$
0	0	1	1		
0	1	1	0		
1	0	0	1		
1	1	0	0		

DeMorgan's Law can be used to convert AND/OR expressions to OR/AND expressions

Example:

$$Z = \overline{A}\,\overline{B}\,C \ + \ \overline{A}\,B\,C \ + \ A\,\overline{B}\,C \ + \ A\,B\,\overline{C}$$

$$\overline{Z} = (A + B + \overline{C}) * (A + \overline{B} + \overline{C}) * (\overline{A} + B + \overline{C}) * (\overline{A} + \overline{B} + C)$$

DeMorgan's Law example

- If $F = (XY+Z)(\overline{Y}+\overline{X}Z)(X\overline{Y}+\overline{Z})$,

$$\overline{F} =$$

NAND and NOR Gates

- NAND Gate: NOT(AND(A, B))

X	Y	X NAND Y
0	0	1
0	1	1
1	0	1
1	1	0

- NOR Gate: NOT(OR(A, B))
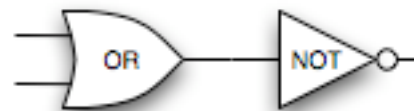
X	Y	X NOR Y
0	0	1
0	1	0
1	0	0
1	1	0

- NAND and NOR gates are universal
  - can implement all the basic gates (AND, OR, NOT)

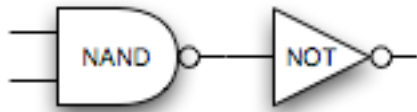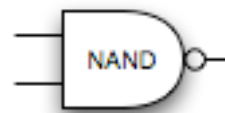NAND   NOR

NOT

AND

OR

Bubble Manipulation

- Bubble Matching
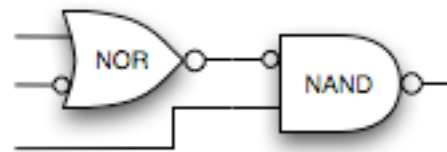
- DeMorgan's Law

XOR and XNOR Gates

- XOR Gate: Z=1 if X is different from Y

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

- XNOR Gate: Z=1 if X is the same as Y

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

Boolean Equations to Circuit Diagrams

- $F = XYZ + \overline{X}Y + XY\overline{Z}$

- $F = XY + X(WZ + \overline{W}\overline{Z})$