# 110
# *Arithmetic*

ENGR 3410 – Computer Architecture

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# Computer Arithmetic

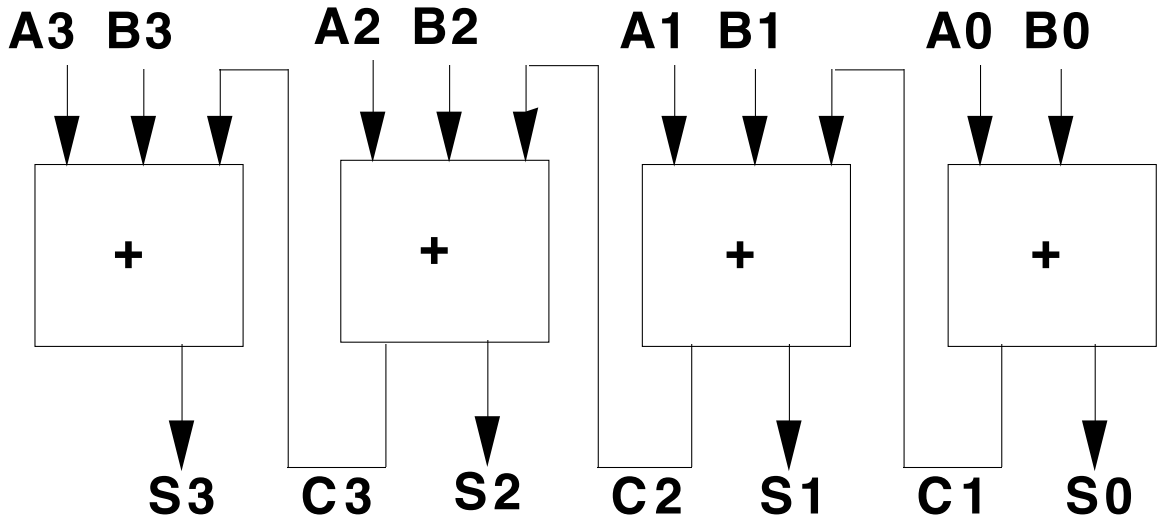
- Readings: Chapter 3
- Review binary numbers, 2's complement
- Develop Arithmetic Logic Units (ALUs) to perform CPU functions.
- Introduce multiplication, division, floating point.

# Full Adder

| A | B | CI | CO | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

# Multi-Bit Addition

Cascaded Multi-bit Adder

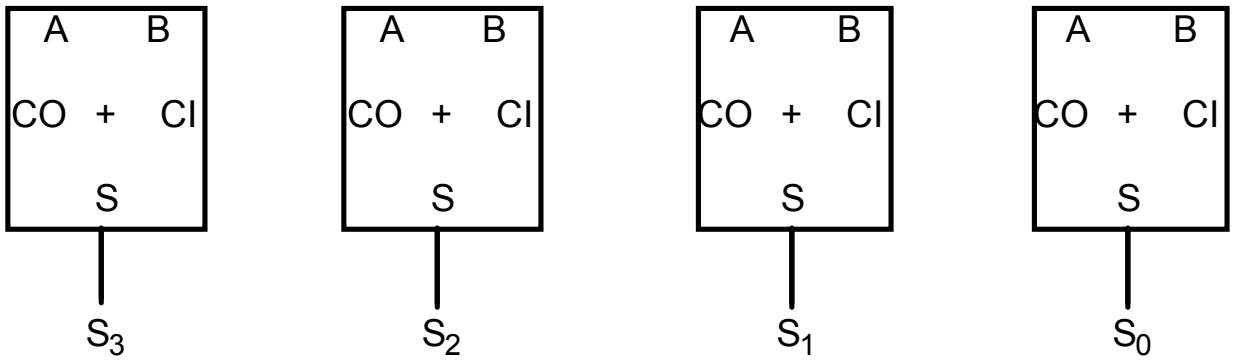



usually interested in adding more than two bits

this motivates the need for the full adder

# Adder/Subtractor

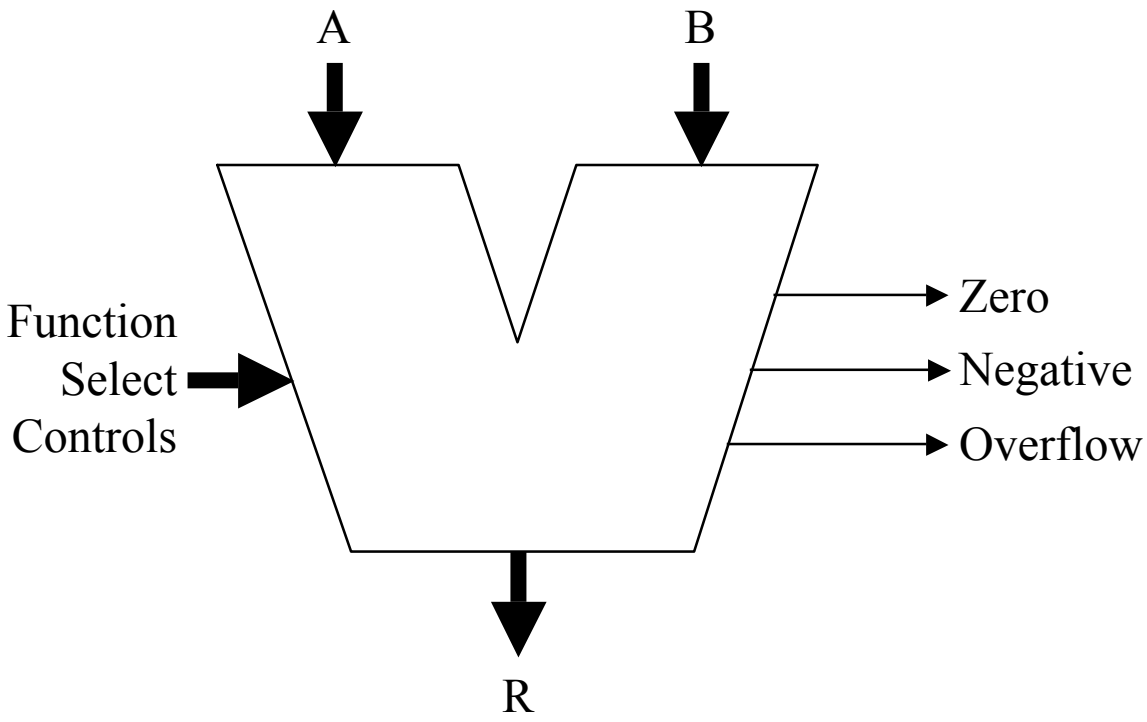
$A_3$ $B_3$ $A_2$ $B_2$ $A_1$ $B_1$ $A_0$ $B_0$




**$A - B = A + (-B) = A + \overline{B} + 1$**

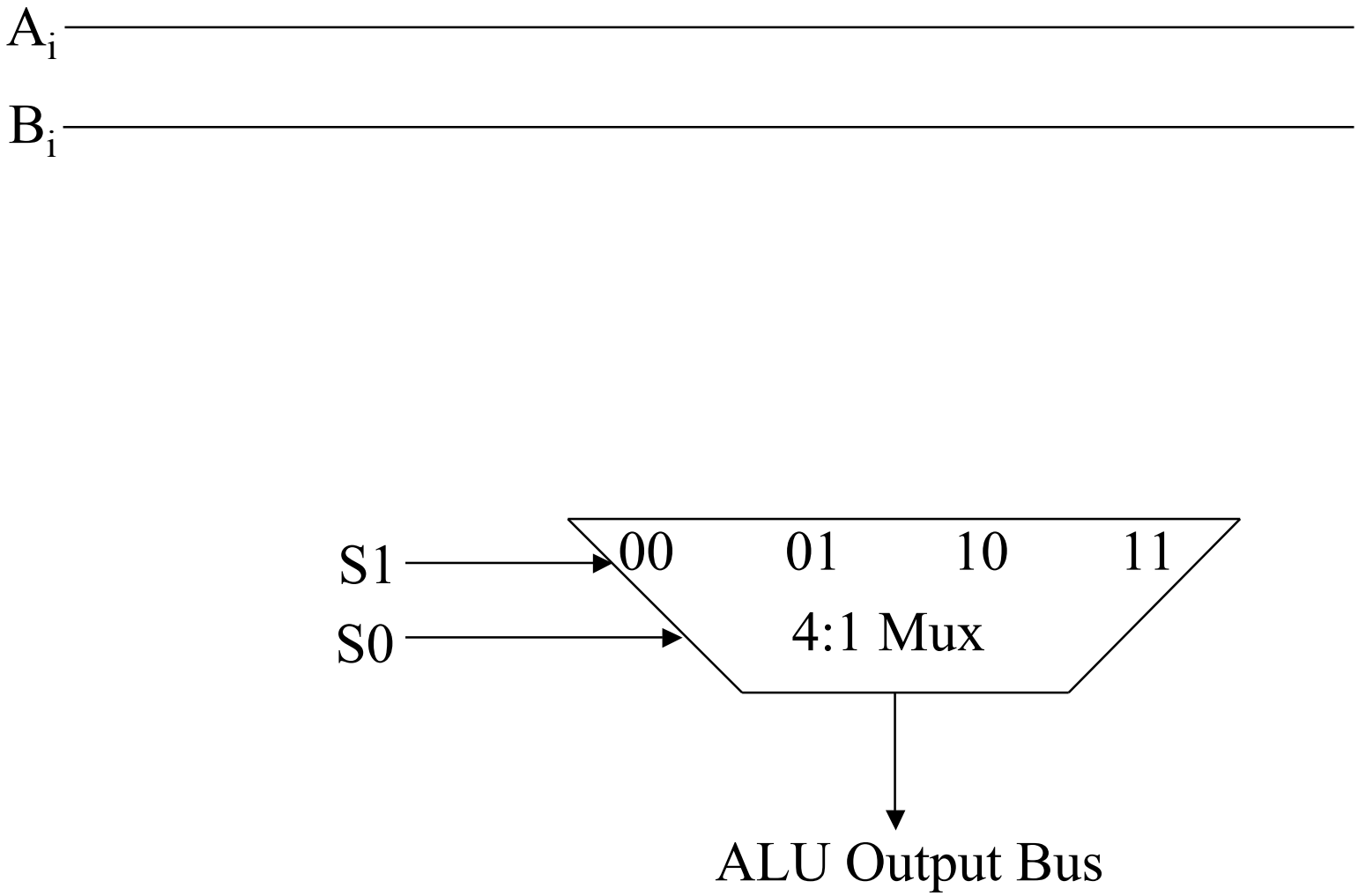
# ALU: Arithmetic Logic Unit

- Computes arithmetic & logic functions based on controls
  - Add, subtract
  - XOR, AND, NAND, OR, NOR
  - ==, <, overflow, …



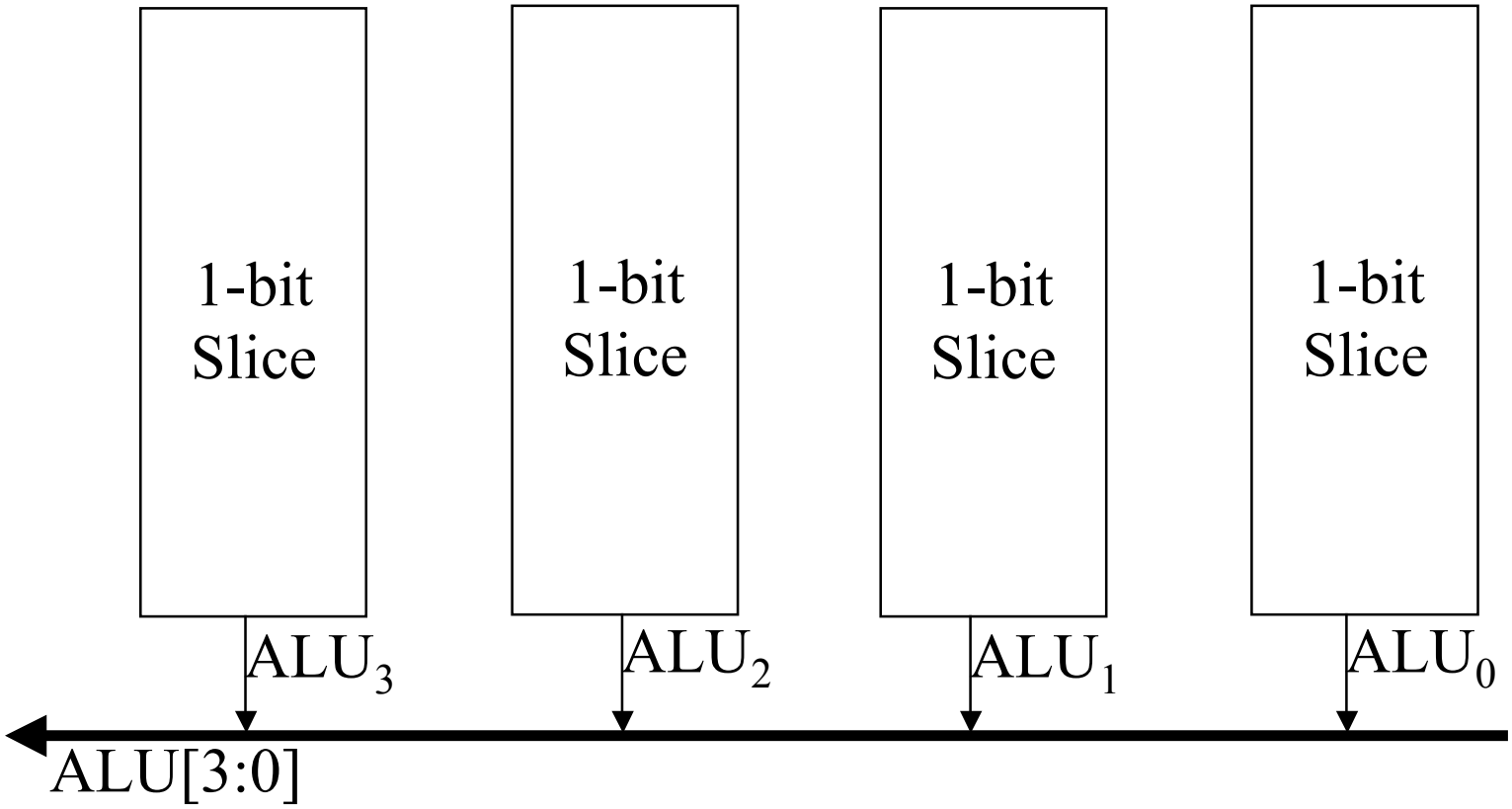
# Bit Slice ALU Design

- Add, Subtract, AND, OR



# Bit Slice ALU Design (cont.)

- Route Carries
- Overflow, zero, negative



# SLT

- Set less than: if (A<B) then R = 1, else R = 0
  - How do we know if (A<B)
  - Interaction w/overflow

# Shifter

- Support shift operations: (A << 01101)
- Optional shift by one: (A << $b_0$)
- Optional shift by two: (A << $b_1$)

# Shifter (cont.)

# Multiplication

- Example

| | | | | | |
|---|---|---|---|---|---|
| **Multiplicand:** | **0** | **1** | **1** | **0** | **6** |
| **Multiplier:** | **0** | **1** | **0** | **1** | **5** |

**4 partial products**

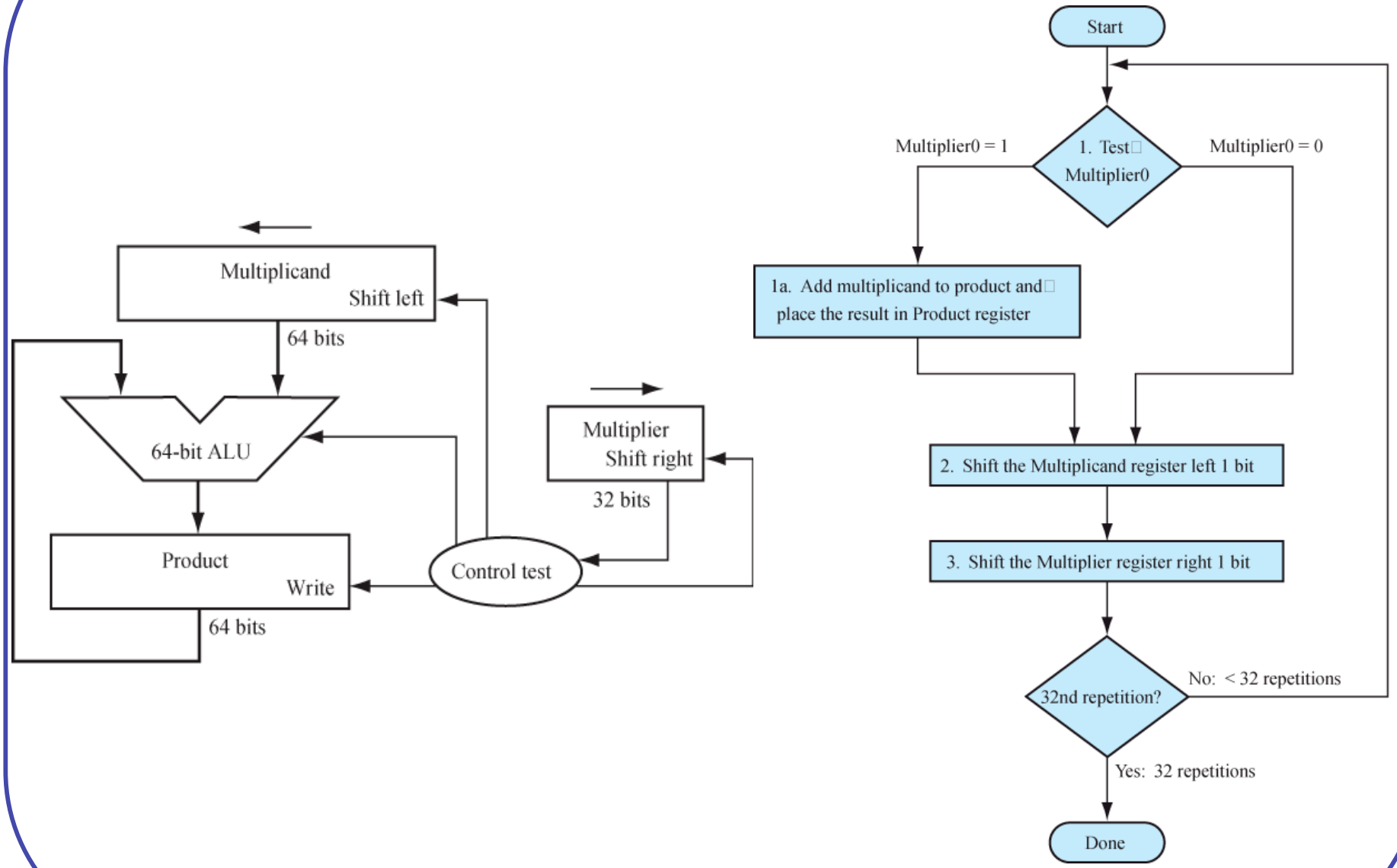
**30**

**Repeat n times:**

**Compute partial product; shift; add**

**NOTE: Each bit of partial products is just an AND operation**

# Sequential Multipliers

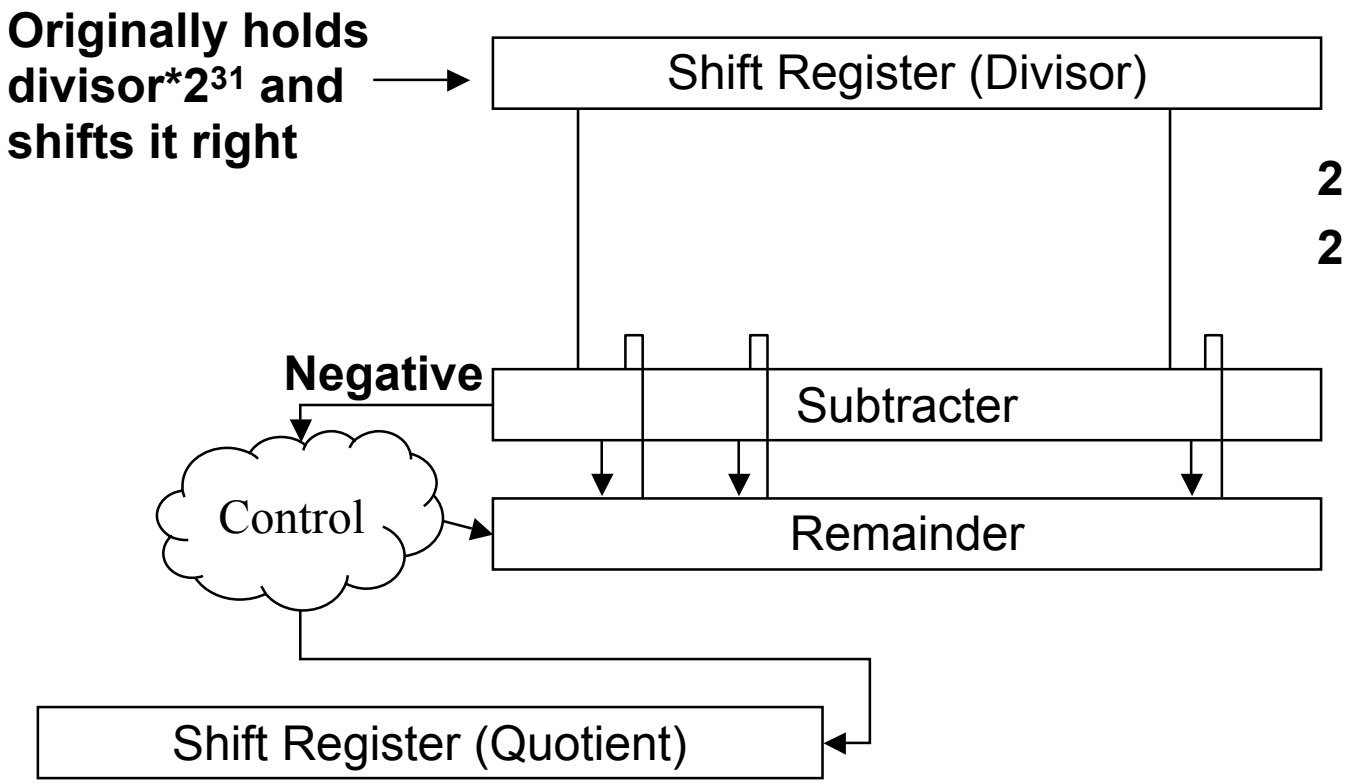


# Division

- Example

**Divisor: 0010 (2)** $\overline{)\ 0\quad 1\quad 1\quad 0}$ **(6) Dividend**

# Sequential Dividers




**Remainder**

**Alternatively:**

**Shift remainder register to left**

**Use only n-bit subtracter**

# Floating Point

Want to represent numbers outside $2^{31}-1$ .. $-2^{31}$

Ideal: ~Scientific Notation

(+/-)Significand*Base$^{\text{Exponent}}$ $5.439 * 10^{12}$ $1.010010 * 2^{100101}$

Multiplication:

$(5.1 * 10^{12}) * (-2.0 * 10^{-3})$

Sign:

Exponent:

Significand:

# Floating Point Addition

Addition: $(5.38 * 10^5) + (4.99 * 10^5)$ $(9.99 * 10^4) + (-1.0 * 10^5)$

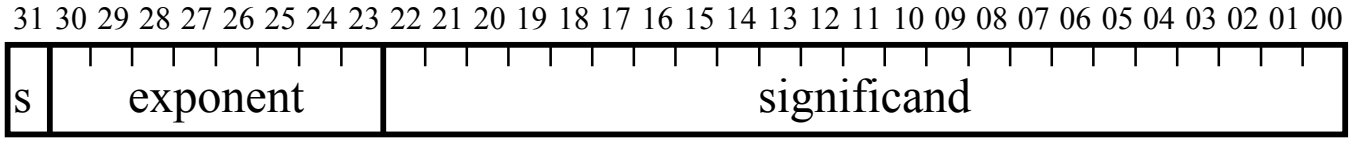
Sign:

Exponent:

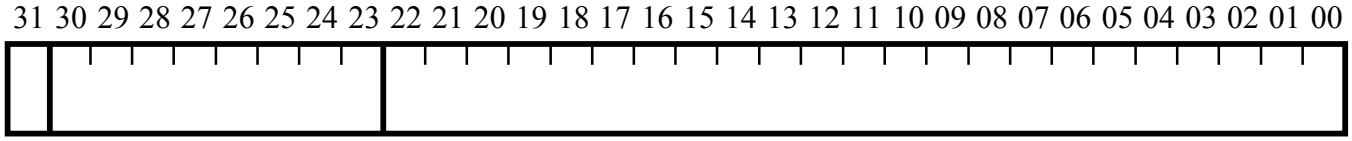
Significand:

# Floating Point Representation

- Floating Point (Float) = $(-1)^s * (1.\text{significand}) * 2^{(\text{exponent}-127)}$

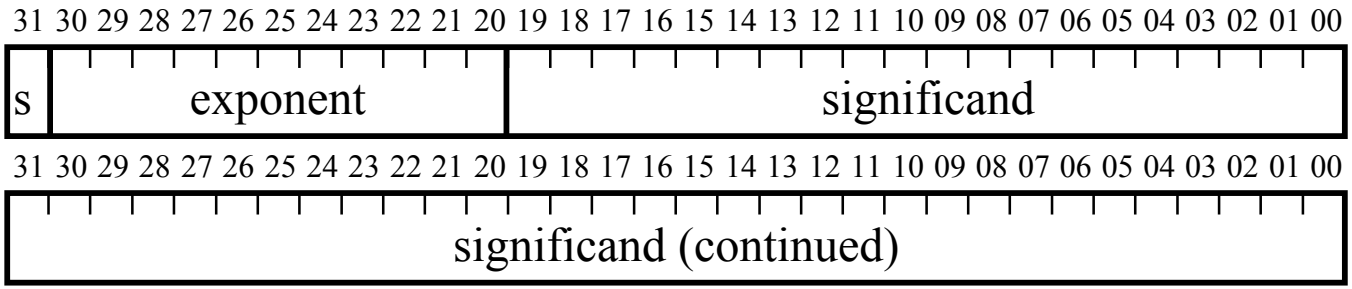



- -0.75 in Floating Point?



# Double Precision Representation

- Double Precision (double) = $(-1)^s$ * (1.significand) * $2^{(exponent-1023)}$




- -0.75 in Double Precision?