10
Number Systems

ENGR 3410 – Computer Architecture

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Decimal (Base 10) Numbers

- Positional system - each digit position has a value

  $2534 = 2*1000 + 5*100 + 3*10 + 4*1$

- Alternate view: Digit position i from the right = Digit * 10^i
  (rightmost is position 0)

  $2534 = 2*10^3 + 5*10^2 + 3*10^1 + 4*10^0$

Base R Numbers

- Each digit in range 0..(R-1)

  0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F ...

  A = 10

  B = 11

  C = 12

  D = 13

  E = 14

  F = 15

- Digit position i = Digit * R^i

  $D_3\ D_2\ D_1\ D_0$ (base R) = $D_3*R^3+D_2*R^2+D_1*R^1+D_0*R^0$

Conversion to Decimal

- Binary: $(101110)_2$

- Octal: $(325)_8$

- Hexadecimal: $(E32)_{16}$

Conversion Decimal

- Binary: $(110101)_2$

- Octal: $(524)_8$

- Hexadecimal: $(A6)_{16}$

Conversion of Decimal to Binary (Method 1)

- For positive, unsigned numbers
- Successively subtract the greatest power of two less than the number from the value. Put a 1 in the corresponding digit position

- $2^0=1$ $2^4=16$ $2^8=256$ $2^{12}=4096$ (4K)
- $2^1=2$ $2^5=32$ $2^9=512$ $2^{13}=8192$ (8K)
- $2^2=4$ $2^6=64$ $2^{10}=1024$ (1K)
- $2^3=8$ $2^7=128$ $2^{11}=2048$ (2K)

Decimal to Binary Method 1

- Convert $(2578)_{10}$ to binary

- Convert $(289)_{10}$ to binary

Conversion of Decimal to Binary (Method 2)

- For positive, unsigned numbers
- Repeatedly divide number by 2. Remainder becomes the binary digits (right to left)
- Explanation:

Decimal to Binary Method 2

- Convert $(289)_{10}$ to binary

Decimal to Binary Method 2

- Convert $(85)_{10}$ to binary

Converting Binary to Hexadecimal

- 1 hex digit = 4 binary digits
- Convert $(11100011010111010011)_2$ to hex

- Convert $(A3FF2A)_{16}$ to binary

Converting Binary to Octal

- 1 octal digit = 3 binary digits
- Convert $(10100101001101010011)_2$ to octal

- Convert $(723642)_8$ to binary

Converting Decimal to Octal/Hex

- Convert to binary, then to other base
- Convert $(198)_{10}$ to Hexadecimal

- Convert $(1983020)_{10}$ to Octal

Arithmetic Operations

Decimal:

$$\begin{array}{r} 5\ 7\ 8\ 9\ 2 \\ +\ 7\ 8\ 9\ 5\ 6 \\ \hline \end{array}$$

Binary:

$$\begin{array}{r} 1\ 0\ 1\ 0\ 1\ 1\ 1 \\ +\ 0\ 1\ 0\ 0\ 1\ 0\ 1 \\ \hline \end{array}$$

Decimal:

$$\begin{array}{r} 5\ 7\ 8\ 9\ 2 \\ -\ 3\ 2\ 9\ 4\ 6 \\ \hline \end{array}$$

Binary:

$$\begin{array}{r} 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0 \\ -\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1 \\ \hline \end{array}$$

Arithmetic Operations (cont.)

Binary:

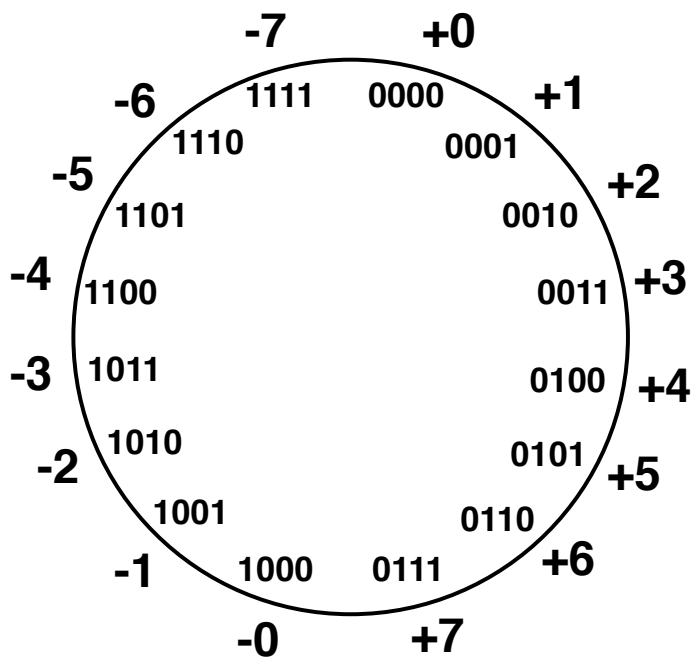
$$\begin{array}{r} 1\ 0\ 0\ 1 \\ *\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

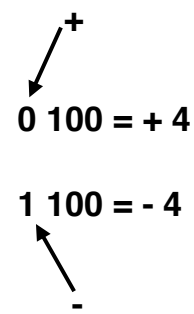
Negative Numbers

- Need an efficient way to represent negative numbers in binary
  - Both positive & negative numbers will be strings of bits
  - Use fixed-width formats (4-bit, 16-bit, etc.)

- Must provide efficient mathematical operations
  - Addition & subtraction with potentially mixed signs
  - Negation (multiply by -1)

Sign/Magnitude Representation





High order bit is sign: 0 = positive (or zero), 1 = negative

Three low order bits is the magnitude: 0 (000) thru 7 (111)

Number range for n bits = $+/-2^{n-1}-1$

Representations for 0:

Sign/Magnitude Addition

Idea: Pick negatives so that addition/subtraction works

$$\begin{array}{rrrrl} & 0 & 0 & 1 & 0 & (+2) \\ + & 0 & 1 & 0 & 0 & (+4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 1 & 0 & 1 & 0 & (-2) \\ + & 1 & 1 & 0 & 0 & (-4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 0 & 0 & 1 & 0 & (+2) \\ + & 1 & 1 & 0 & 0 & (-4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 1 & 0 & 1 & 0 & (-2) \\ + & 0 & 1 & 0 & 0 & (+4) \\ \hline \end{array}$$

Bottom line: Basic mathematics are too complex in Sign/Magnitude

Idea: Pick negatives so that addition works

- Let -1 = 0 - (+1):

$$\begin{array}{rrrrr} 0 & 0 & 0 & 0 & (\ 0) \\ -\ 0 & 0 & 0 & 1 & (+1) \\ \hline \end{array}$$

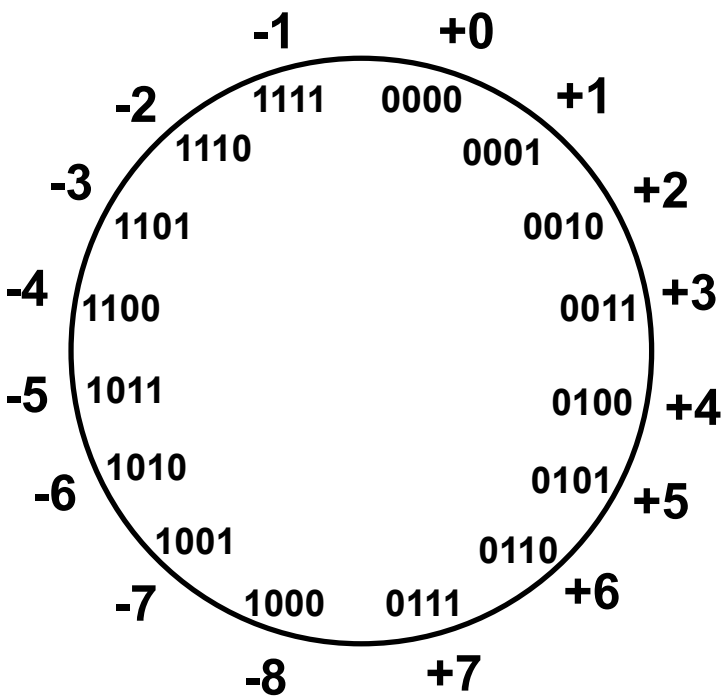
- Does addition work?

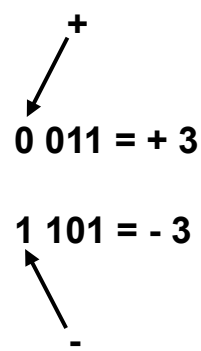
$$\begin{array}{rrrrr} 0 & 0 & 1 & 0 & (+2) \\ +\ 1 & 1 & 1 & 1 & (\ -1) \\ \hline \end{array}$$

- Result: Two's Complement Numbers

Two's Complement

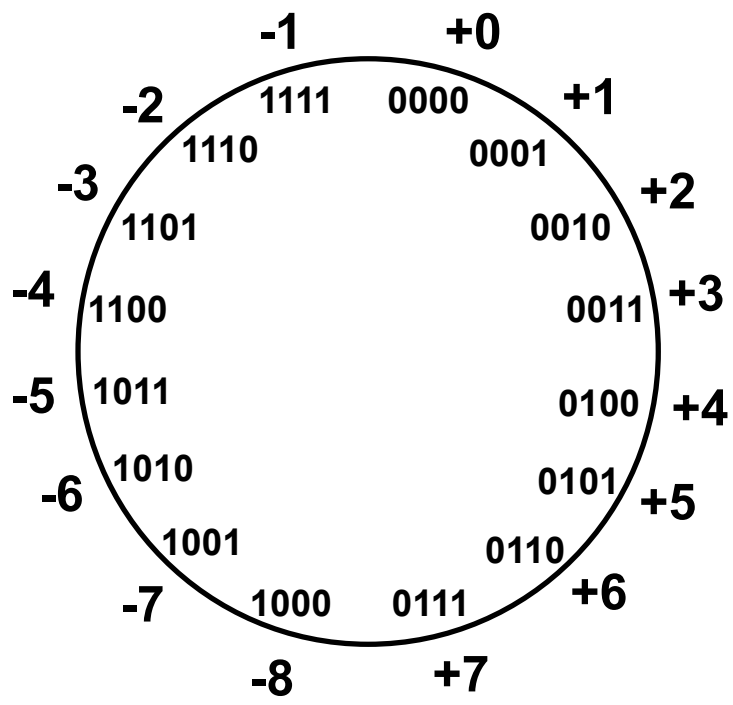
- Only one representation for 0
- One more negative number than positive number
- Fixed width format for both pos. & neg. numbers





Negating in Two's Complement

- Flip bits & Add 1
- Negate $(0010)_2$ (+2)
- Negate $(1110)_2$ (-2)



Addition in Two's Complement

$$\begin{array}{rrrrl} & 0 & 0 & 1 & 0 \;\; (+2) \\ + & 0 & 1 & 0 & 0 \;\; (+4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 1 & 1 & 1 & 0 \;\; (-2) \\ + & 1 & 1 & 0 & 0 \;\; (-4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 0 & 0 & 1 & 0 \;\; (+2) \\ + & 1 & 1 & 0 & 0 \;\; (-4) \\ \hline \end{array}$$

$$\begin{array}{rrrrl} & 1 & 1 & 1 & 0 \;\; (-2) \\ + & 0 & 1 & 0 & 0 \;\; (+4) \\ \hline \end{array}$$

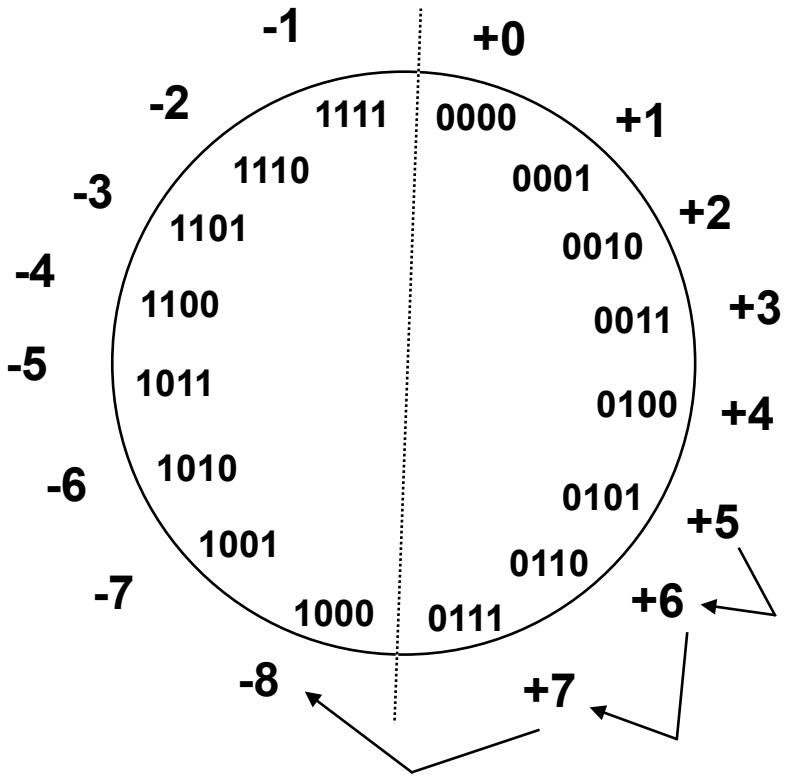
Subtraction in Two's Complement

- $A - B = A + (-B) = A + \overline{B} + 1$
- 0010 - 0110
- 1011 - 1001
- 1011 - 0001

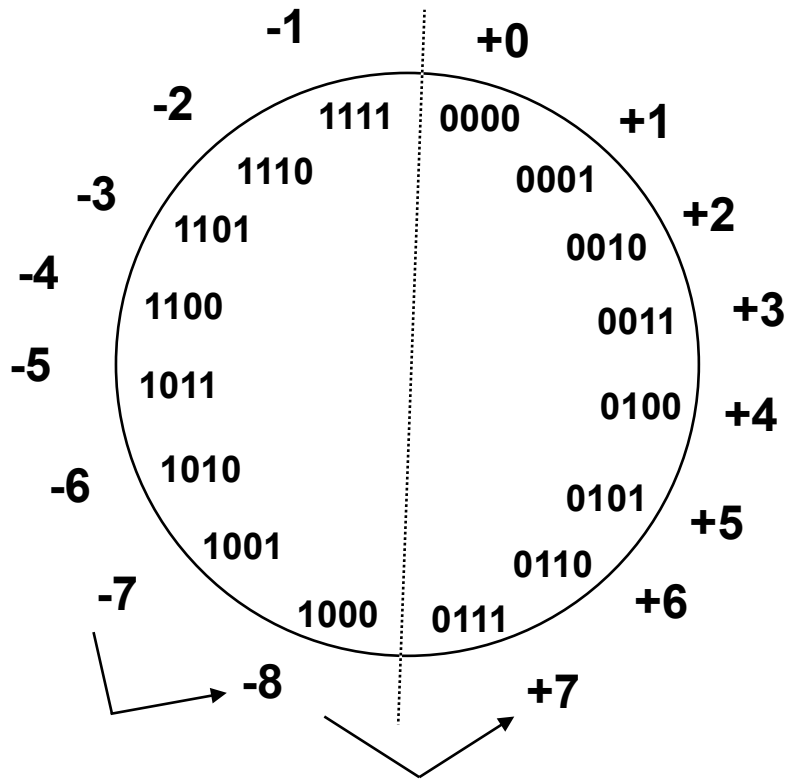
Overflows in Two's Complement

Add two positive numbers to get a negative number

or two negative numbers to get a positive number



5 + 3 = -8



-7 - 2 = +7

Overflow Detection in Two's Complement

5	0 1 0 1
3	0 0 1 1
-8	

Overflow

-7	1 0 0 1
-2	1 1 1 0
7	

Overflow

5	0 1 0 1
2	0 0 1 0
7	

No overflow

-3	1 1 0 1
-5	1 0 1 1
-8	

No overflow

Overflow when carry in to sign does not equal carry out

Converting Decimal to Two's Complement

- Convert absolute value to binary, then negate if necessary
- Convert $(-9)_{10}$ to 6-bit Two's Complement
- Convert $(9)_{10}$ to 6-bit Two's Complement

Converting Two's Complement to Decimal

- If Positive, convert as normal;
  If Negative, negate then convert.

- Convert $(11010)_2$ to Decimal

- Convert $(01011)_2$ to Decimal

Sign Extension

- To convert from N-bit to M-bit Two's Complement (N>M), simply duplicate sign bit:

- Convert $(1011)_2$ to 8-bit Two's Complement

- Convert $(0010)_2$ to 8-bit Two's Complement