Practice Problems 1 Solutions

Full Adder Sum

We already covered the full adder on day 1 and found that the 'sum' output is equal to 'A xor B xor C'. Use the identity 'X xor Y = ~X&Y + X&~Y' and DeMorgan's law to expand this to a sum of products notation.

Be careful! On the second expansion of the XOR identity it is very easy to get the parenthesis wrong.

Check your resulting equation's truth table against the original truth table to verify.

Process

1. $A \boxplus B \boxplus Cin$	Starting equation
2. $(\bar{A}B + A\bar{B}) \boxplus Cin$	First XOR expansion
3. $(\bar{A}B + A\bar{B})\bar{C} + \overline{(\bar{A}B + A\bar{B})}C$	Second XOR expansion (Cin -> C)
4. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + \overline{(\bar{A}B + A\bar{B})}C$	Distribute ~C on left
5. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + (\overline{\bar{A}B})(\overline{A\bar{B}})C$	DeMorgan's Law
6. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + (A + \bar{B})(\bar{A} + B)C$	DeMorgan's Law again
7. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + (A\bar{A} + AB + \bar{B}\bar{A} + \bar{B}B)C$	Distribute
8. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + (0 + AB + \bar{B}\bar{A} + 0)C$	X*~X = 0
9. $\bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}\bar{B}C$	Distribute

Full Adder Carry

Convert the carry logic for a full adder to use all NAND gates. Remember, an Inverter is just a 1 input NAND gate (or an N-input NAND gate with its legs tied together).

Process

The Karnaugh Map gives us F= AB+AC+BC. This is represented by 3 2-input AND gates and 1 3-input OR gate. Applying DeMorgan's transformation to the OR gate transforms it to a NAND gate with inverters on all of its inputs. Push the inverters to the remaining AND gates to make them NAND gates. The result uses 1 3-input NAND and 3 2-input NANDs.

$$AB + AC + BC \rightarrow \overline{(\overline{AB})(\overline{AC})(\overline{BC})}$$